

Cosmology and the origin of structure

<http://home.fnal.gov/~rocky/>

Rocky I: The universe observed

Rocky II: The growth of cosmological structures

Rocky III: Inflation and the origin of perturbations-1

Rocky IV: Inflation and the origin of perturbations-2

Rocky V: Dark matter

Xth Brazilian School of Cosmology and Gravitation

August 2002, Mangaratiba

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Fermilab, University of Chicago, & CERN

Issues

- 1. Transplanckian physics**
 - probe of short-distance physics?
- 2. Defrosting**
 - preheating, reheating,
- 3. Particle production**
 - WIMPZILLAS, gravitons,
- 4. Why only one inflaton?**
 - isocurvature perturbations
- 5. Extra dimensions, brane, bulk, etc.?**
 - new dynamics

The vacuum

Mode equation:

$$\frac{d^2 u_k}{d\tau^2} + \left(k^2 - \frac{1}{z} \frac{d^2 z}{d\tau^2} \right) u_k = 0 \quad (z = a\phi'_0 / H)$$

$$\frac{1}{z} \frac{d^2 z}{d\tau^2} = 2a^2 H^2 \times (\text{slow-roll parameters})$$

Infrared $k \rightarrow 0$ limit:

$$\frac{u''_k}{u_k} = \frac{z''}{z} \Rightarrow \frac{u_k}{z} = \mathfrak{R}_k = \text{constant}$$



Ultraviolet $k \rightarrow \infty$ limit:

$$u''_k + k^2 u_k = 0 \Rightarrow u_k = \text{constant} \times \exp(-ik\tau)$$

The vacuum

$$\frac{d^2 u_k}{d\tau^2} + \left(k^2 - a^2 H^2 \dots \right) u_k = 0$$

Solutions are Bessel functions: need boundary conditions

$$u_k \rightarrow \frac{1}{\sqrt{2k}} \exp(-ik\tau) \quad \text{as } \frac{k}{aH} \rightarrow \infty$$

$$\rightarrow u_k(\tau) = \frac{\sqrt{\pi}}{2} e^{i(\nu+1/2)\pi/2} (-\tau)^{1/2} H_\nu^{(1)}(-k\tau)$$

Why assume $u_k \rightarrow \frac{1}{\sqrt{2k}} \exp(-ik\tau)$?

The vacuum

$$u(\tau, \vec{x}) = \int d^3k \left[u_{\vec{k}}(\tau) a_{\vec{k}} e^{i\vec{k} \cdot \vec{x}} + u_{\vec{k}}^*(\tau) a_{\vec{k}}^\dagger e^{-i\vec{k} \cdot \vec{x}} \right]$$

Why assume $u_{\vec{k}}(\tau) \rightarrow \frac{1}{\sqrt{2k}} \exp(-ik\tau)$?

- Bunch-Davies vacuum-positive frequency modes
- Associated operator $a_{\vec{k}}$ annihilates the Bunch-Davies vacuum $a_{\vec{k}} |0\rangle_{\text{BD}} = 0$
- Could make another choice for mode functions with associated operators $b_{\vec{k}}$ and vacuum $b_{\vec{k}} |0\rangle_{\text{N}} = 0$

$$u(\tau, \vec{x}) = \int d^3k \left[v_{\vec{k}}(\tau) b_{\vec{k}} e^{i\vec{k} \cdot \vec{x}} + v_{\vec{k}}^*(\tau) b_{\vec{k}}^\dagger e^{-i\vec{k} \cdot \vec{x}} \right]$$

The vacuum

Choices for mode functions and annihilation operators

$$\begin{aligned} u(\tau, \vec{x}) &= \int d^3k \left[u_{\vec{k}}(\tau) a_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} + u_{\vec{k}}^*(\tau) a_{\vec{k}}^\dagger e^{-i\vec{k}\cdot\vec{x}} \right] \quad \text{B.D.} \\ &= \int d^3k \left[v_{\vec{k}}(\tau) b_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} + v_{\vec{k}}^*(\tau) b_{\vec{k}}^\dagger e^{-i\vec{k}\cdot\vec{x}} \right] \quad \text{N.} \end{aligned}$$

Mode functions and annihilation operators related

$$v_{\vec{k}}(\tau) = \alpha_k u_{\vec{k}}(\tau) + \beta_k u_{\vec{k}}^*(\tau)$$

$$b_{\vec{k}} = \alpha_k^* a_{\vec{k}} + \beta_k^* a_{\vec{k}}^\dagger$$

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

The vacuum

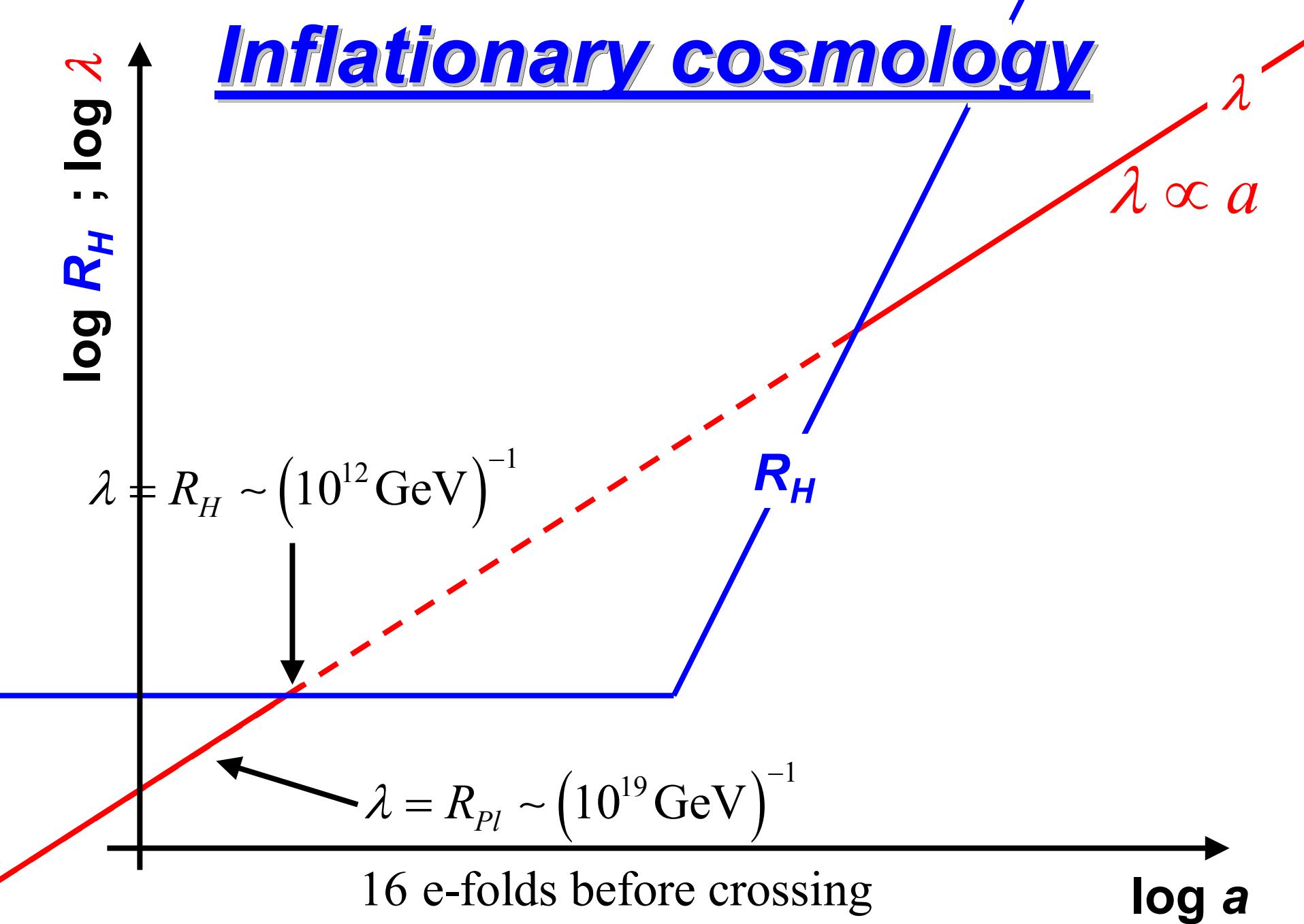
$${}_N\langle 0 | \frac{u(\tau, \vec{x}_1)}{z} \frac{u(\tau, \vec{x}_2)}{z} | 0 \rangle_N \rightarrow (\mathcal{R}_k)_N$$

$$(\mathcal{R}_k)_N \rightarrow \left[|\alpha_k|^2 + |\beta_k|^2 + 2 \operatorname{Re}(\alpha_k \beta_k^*) \right] (\mathcal{R}_k)_{\text{BD}}$$

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

- The particle number density proportional to $|\beta_k|^2$
- There might be new physics that changes vacuum at scales less than Hubble radius!

Inflationary cosmology



The vacuum

- If you cannot support imposing boundary conditions at $k = \infty$, then impose vacuum at scale $k = \Lambda$
- Then for instance for gravitons

$$P_g = \left(\frac{H}{2\pi} \right)^2 \left[1 - \frac{H}{\Lambda} \right]$$

- Recall

$$\left(\frac{H}{m_{PL}} \right) \leq 10^{-6}$$

The vacuum

$$(\mathcal{R}_k)_N \rightarrow \left[|\alpha_k|^2 + |\beta_k|^2 + 2 \operatorname{Re}(\alpha_k \beta_k^*) \right] (\mathcal{R}_k)_{BD}$$

- Scales we observe today were once *transplanckian*
- Transplanckian effects may result in particle creation
- Particles in vacuum at early time change result
- But they also contribute to energy-momentum tensor
- Large enough effect to modify result bad for inflation
- If define vacuum at scale Λ

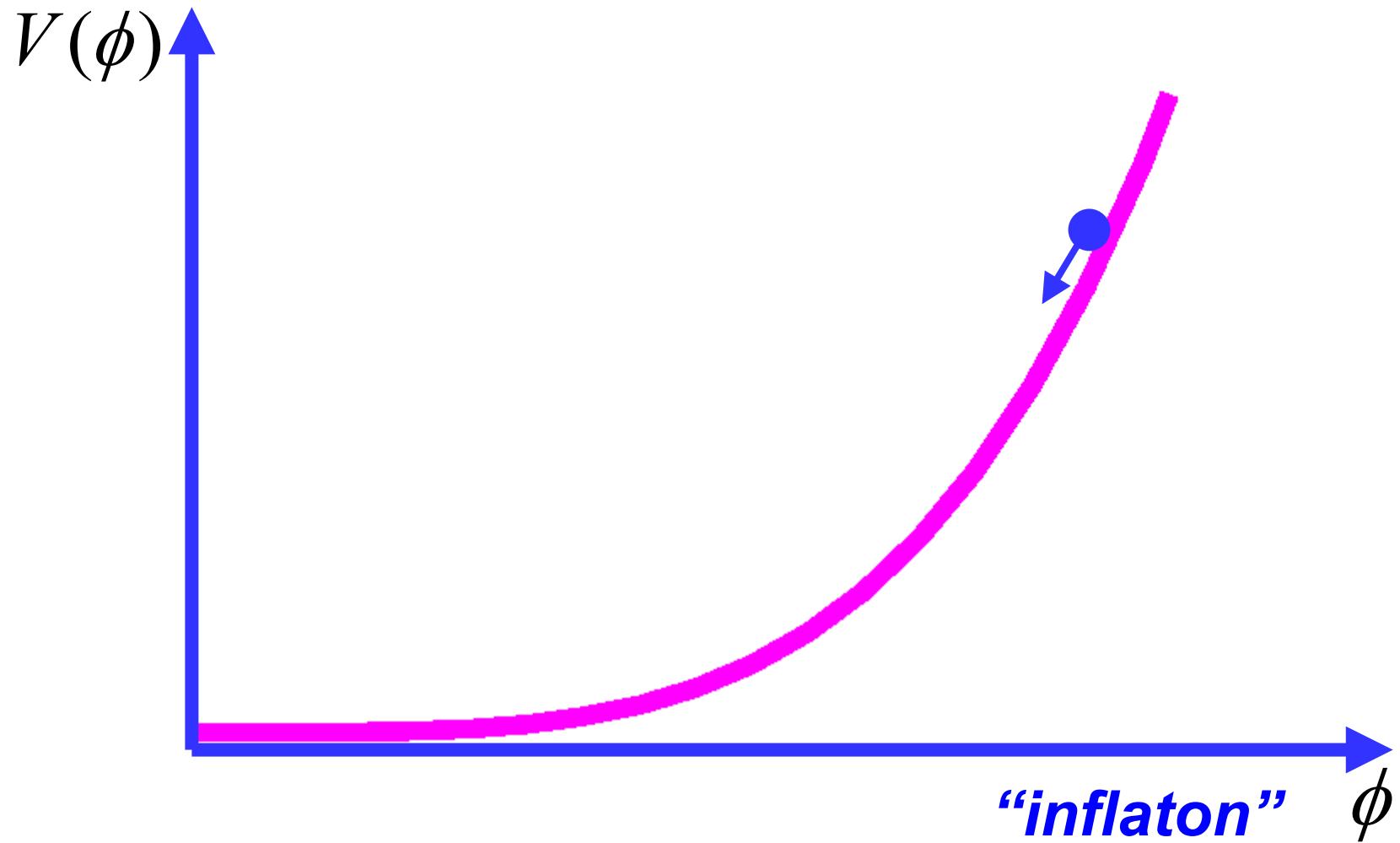
$$|\beta_k| \leq \frac{Hm_{Pl}}{\Lambda^2}$$

The vacuum

$$(\mathcal{R}_k)_N \rightarrow \left[|\alpha_k|^2 + |\beta_k|^2 + 2 \operatorname{Re}(\alpha_k \beta_k^*) \right] (\mathcal{R}_k)_{BD}$$

- Simple result seems robust
- An effect is possible for physics scales greater than Planck length (but smaller than Hubble radius)
 - string scale
 - scale of extra dimensions
- Possible non-Gaussian effects (Gangui)

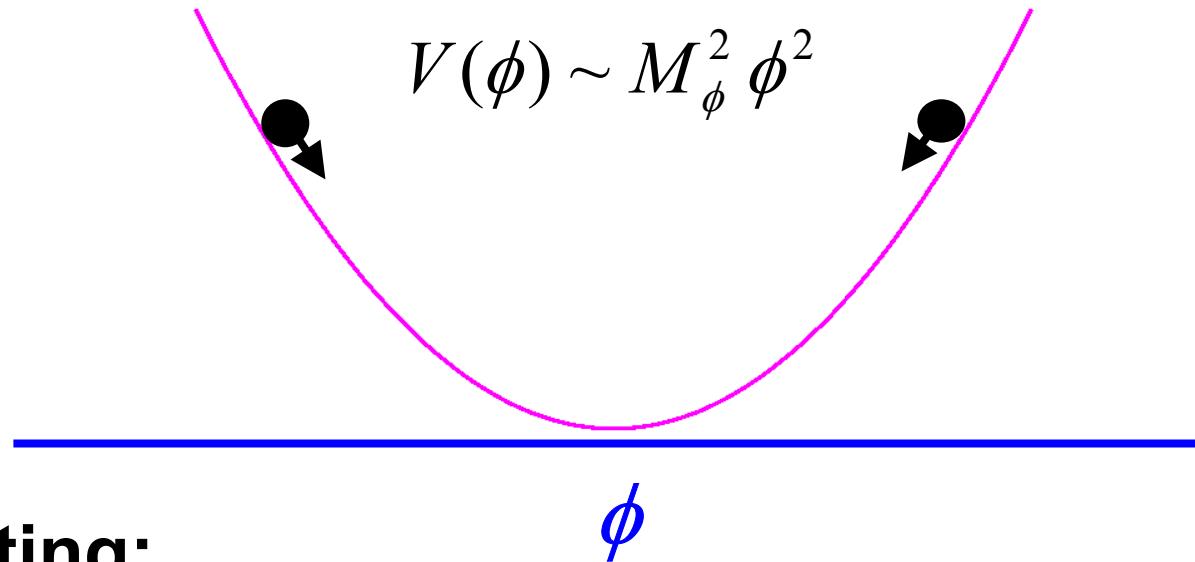
ca. 1981-1983: Guth; Albrecht & Steinhardt; Linde



Classical equations of motion
 $V(\phi) \neq 0 \longrightarrow V(\phi) = 0$

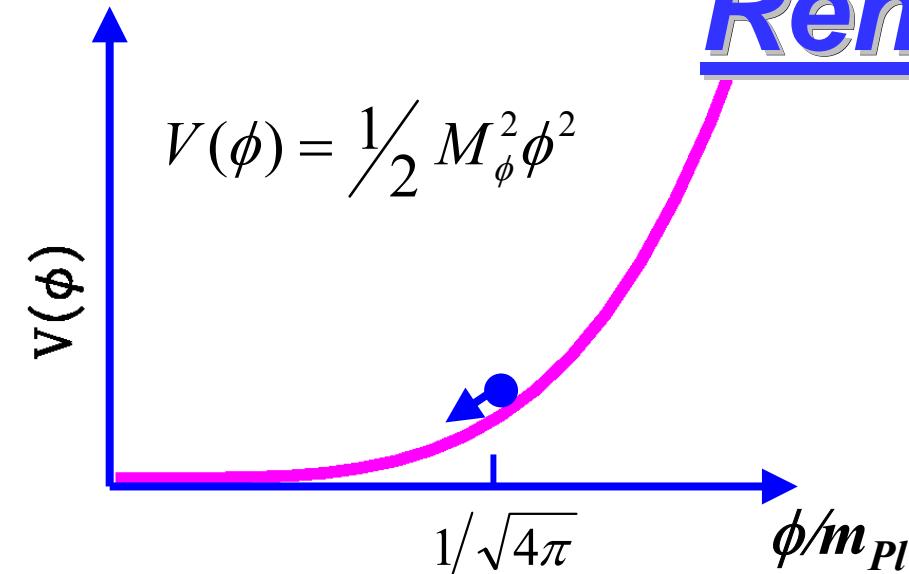
The end of inflation

- after inflation the universe is frozen



- defrosting:
 - reheating (ca. early 80s)
incoherent, nonresonant, linear decay of inflaton
 - preheating (ca. mid 90s)
coherent, resonant, nonlinear particle production

Reheating



$$M_\phi = 10^{13} \text{ GeV}$$

$$\phi_{END} = m_{Pl} / \sqrt{4\pi}$$

- coherent ϕ oscillations with decreasing amplitude

$$\omega = M_\phi \quad \rho_\phi \propto a^{-3}$$

- ϕ coupled to other fields

ρ_ϕ decays with width Γ_ϕ

- decay produces massless degrees of freedom (γ)
thermalize to temperature T

- when “all” energy extracted from ϕ , $T=T_{RH}$
gravitino limit $T_{RH} < 10^9 \text{ GeV}$

Reheating

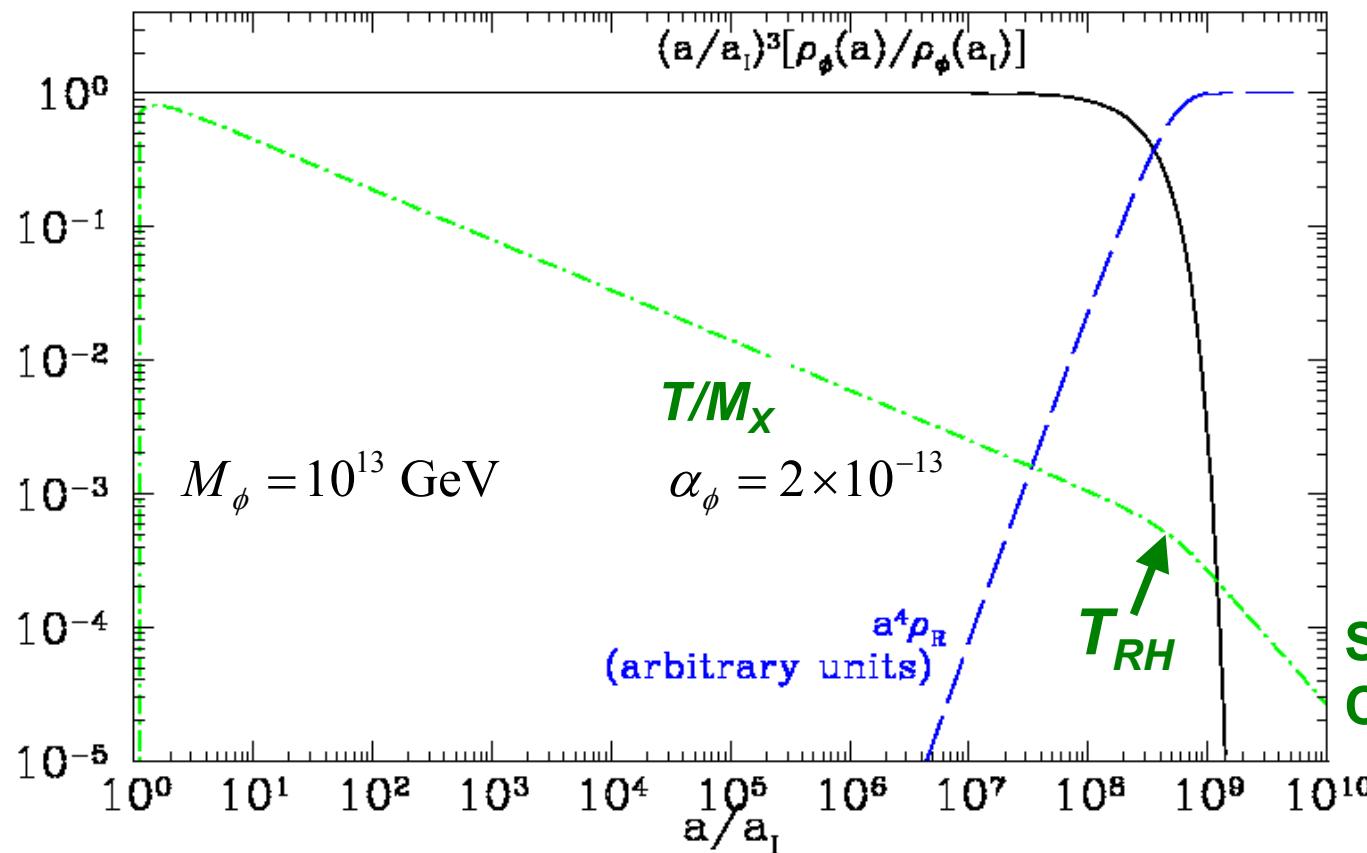
$$\phi = \text{inflaton} \quad \Gamma_\phi = \alpha_\phi M_\phi$$

$$\dot{\rho}_\phi + 3H\rho_\phi + \Gamma_\phi \rho_\phi = 0$$

$$\dot{\rho}_R + 4H\rho_R - \Gamma_\phi \rho_\phi = 0$$

$$\Omega_X h^2 \propto \exp(-M_X/T_{RH})$$

$$\Omega_X h^2 \propto (2000 T_{RH}/M_X)^7$$



$$T_{MAX} \gg T_{RH}$$

Scherrer & Turner
Chung, Kolb, & Riotto

Preheating

- Inflaton ϕ coupled to another scalar field χ :

$$V = \frac{1}{2} g^2 \phi^2 \chi^2$$

- Field equation:

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{\vec{k}^2}{a^2} + m_\chi^2 - \cancel{\varepsilon R} + g^2 \phi^2 \right) \chi_k = 0$$

\vec{k} = comoving momentum $\phi(t) = \Phi(t) \sin mt$

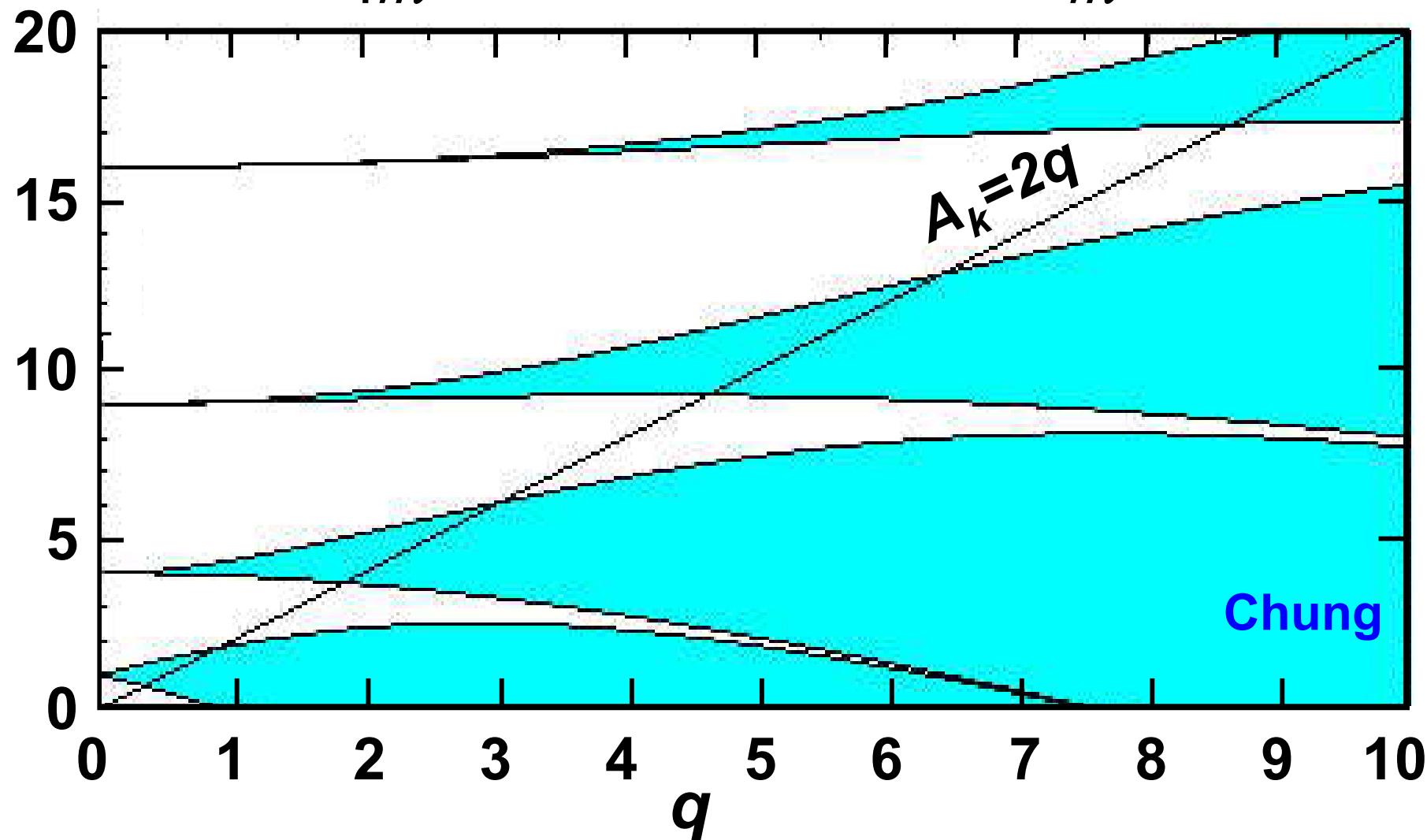
- Minkowski space Φ constant: Matheiu equation

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{\vec{k}^2}{a^2} + g^2 \Phi^2 \sin^2 mt \right) \chi_k = 0$$

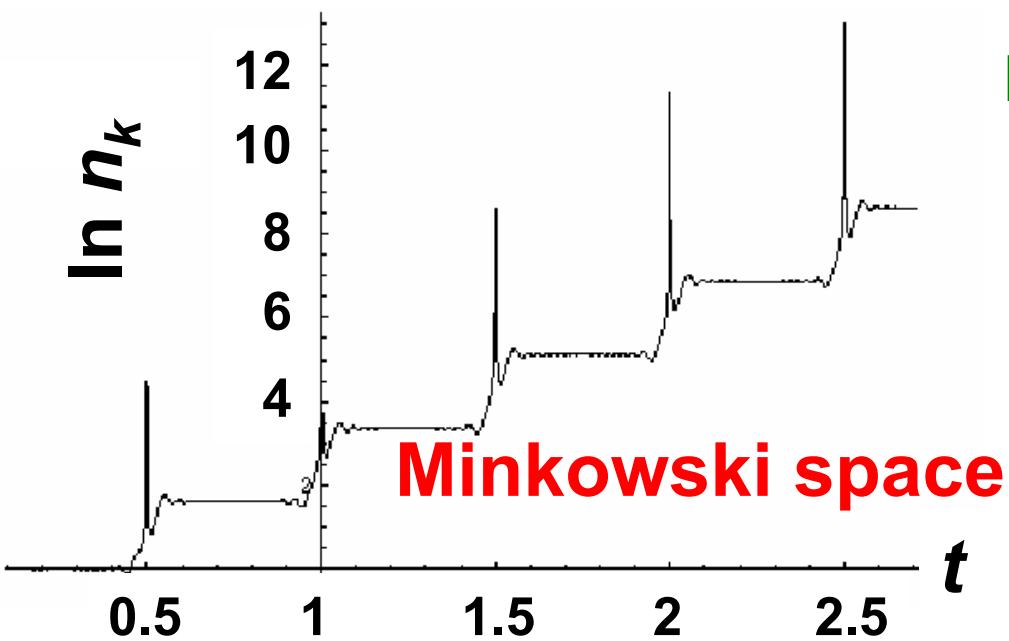
Instability regions

$$q = \frac{g^2 \Phi^2}{4m^2}$$

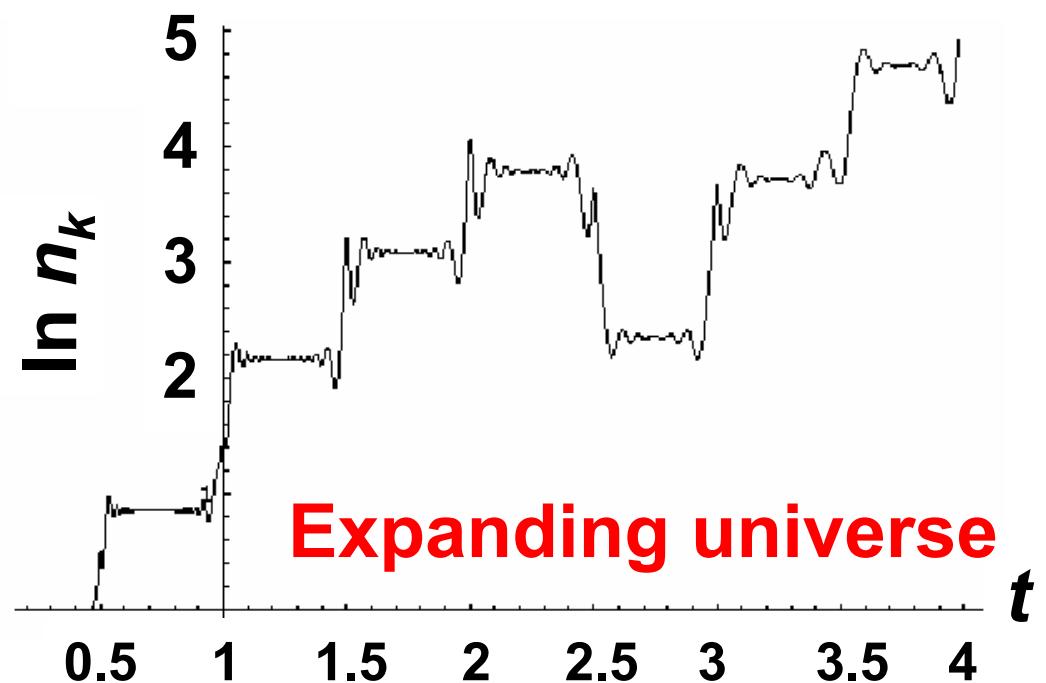
$$A_k = 2q + \frac{k^2}{m^2}$$



Kofman, Linde, Starobinski
Phys. Rev. D (1997)

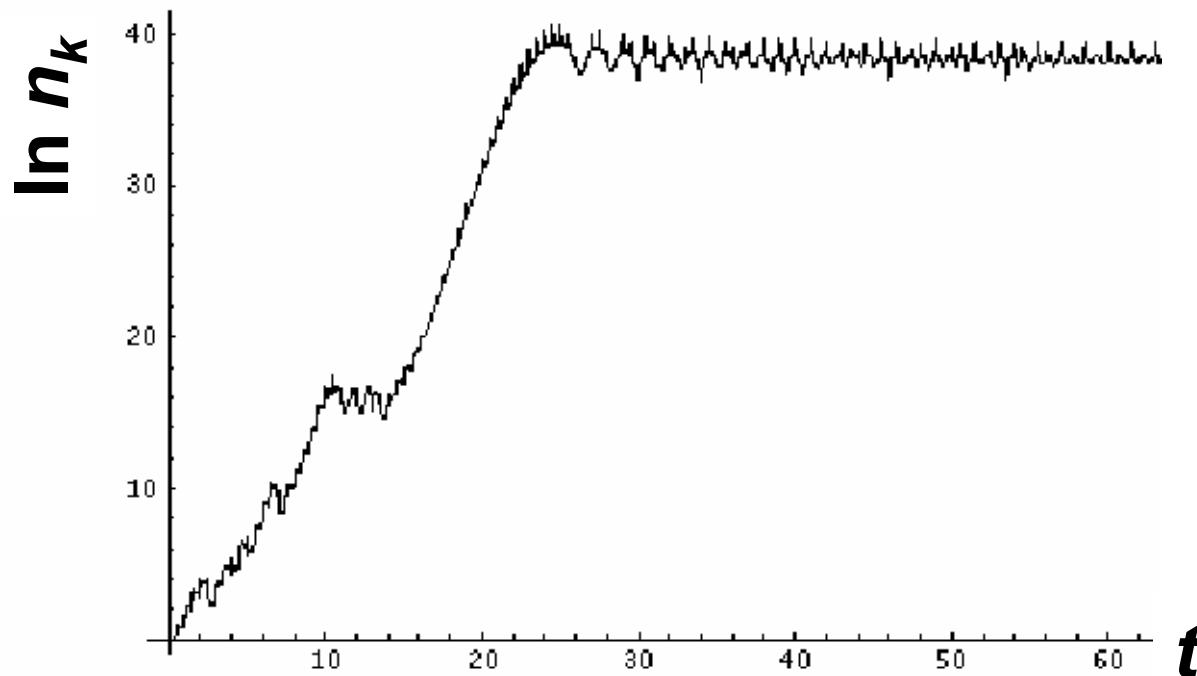


Minkowski space



Expanding universe

**Kofman, Linde, Starobinski
Phys. Rev. D (1997)**

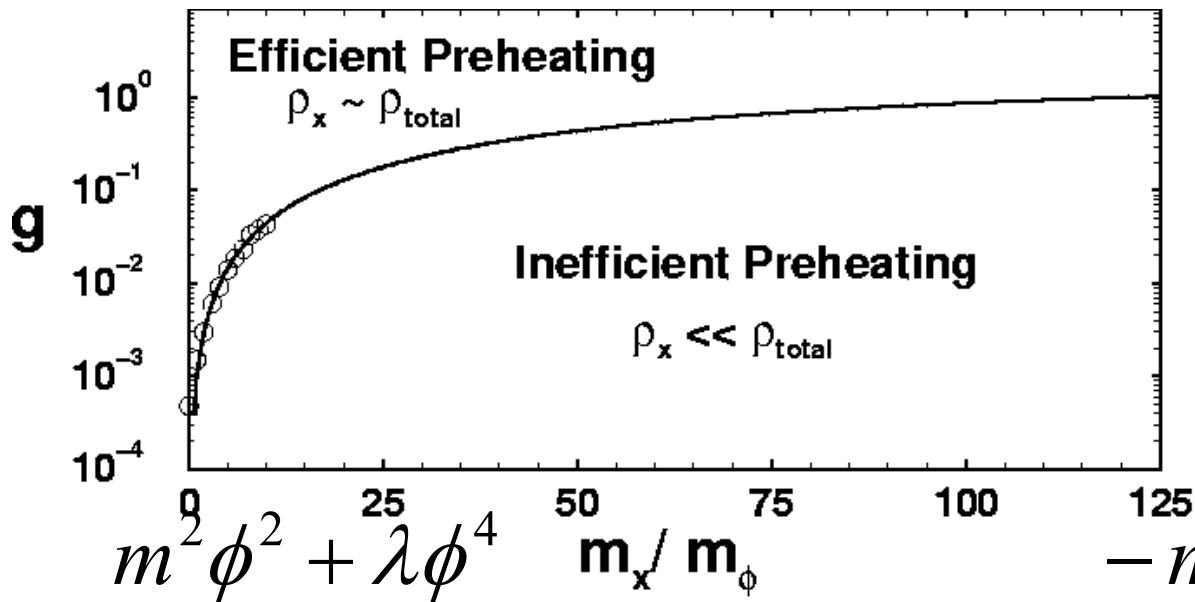


Complexity: rescatterings, back reactions, ...

**Conclusion: in a few dozen oscillation periods,
significant fraction of energy extracted**

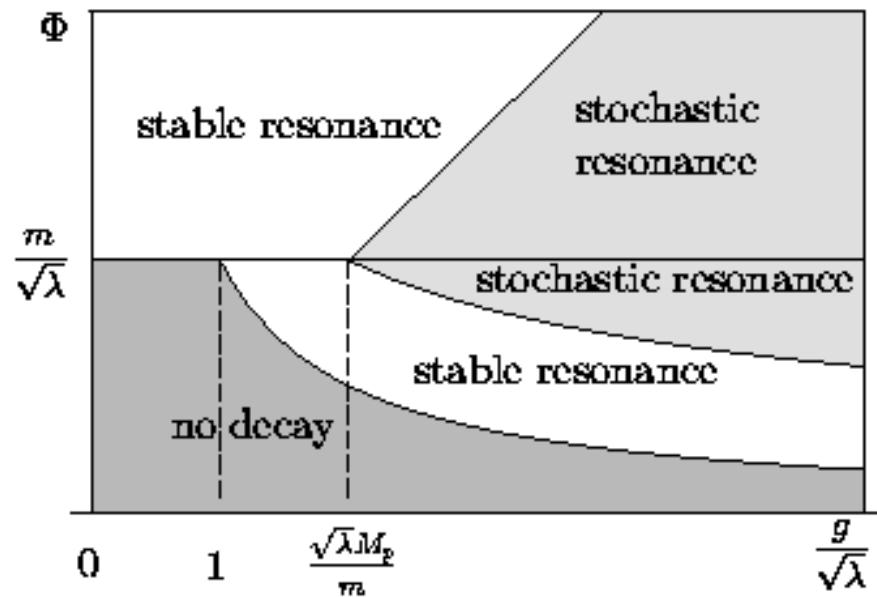
**Note: inherently coherent process-
many inflaton quanta participate**

Parameter Space

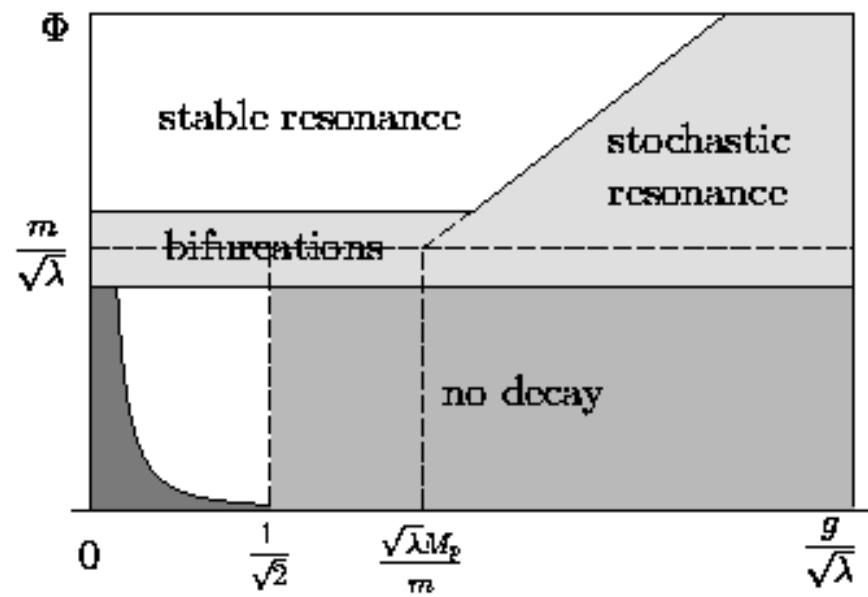


$$g^2 \phi^2 \chi^2$$

$$-m^2 \phi^2 + \lambda \phi^4$$



Greene et al.



Preheating/reheating issues

- Preheating or reheating?
- What is the “reheat” temperature?
- Symmetry restoration?
- Massive particle production?
 - *baryogenesis/leptogenesis*
 - *dark matter*
- Light particle production?
 - *gravitinos*
- Perturbations?
 - *curvature*
 - *isocurvature*

Expanding universe → particle creation

Arnowit, Birrell, Bunch, Davies, Deser, Ford, Fulling, Grib, Hu, Kofman, Lukash, Mostepanenko, Page, Parker, Starobinski, Unruh, Vilenkin, Wald, Zel'dovich,...

It's not a bug, it's a feature!

first application: { density perturbations from inflation
gravitational waves from inflation

(Guth & Pi; Starobinski; Bardeen, Steinhardt, & Turner; Hawking; Rubakov; Fabbi & Pollack; Allen)

new application: **dark matter**

(Chung, Kolb, & Riotto; Kuzmin & Tkachev)

- require (super)massive particle “X”
- stable (or at least long lived)
- initial inflationary era followed by radiation/matter

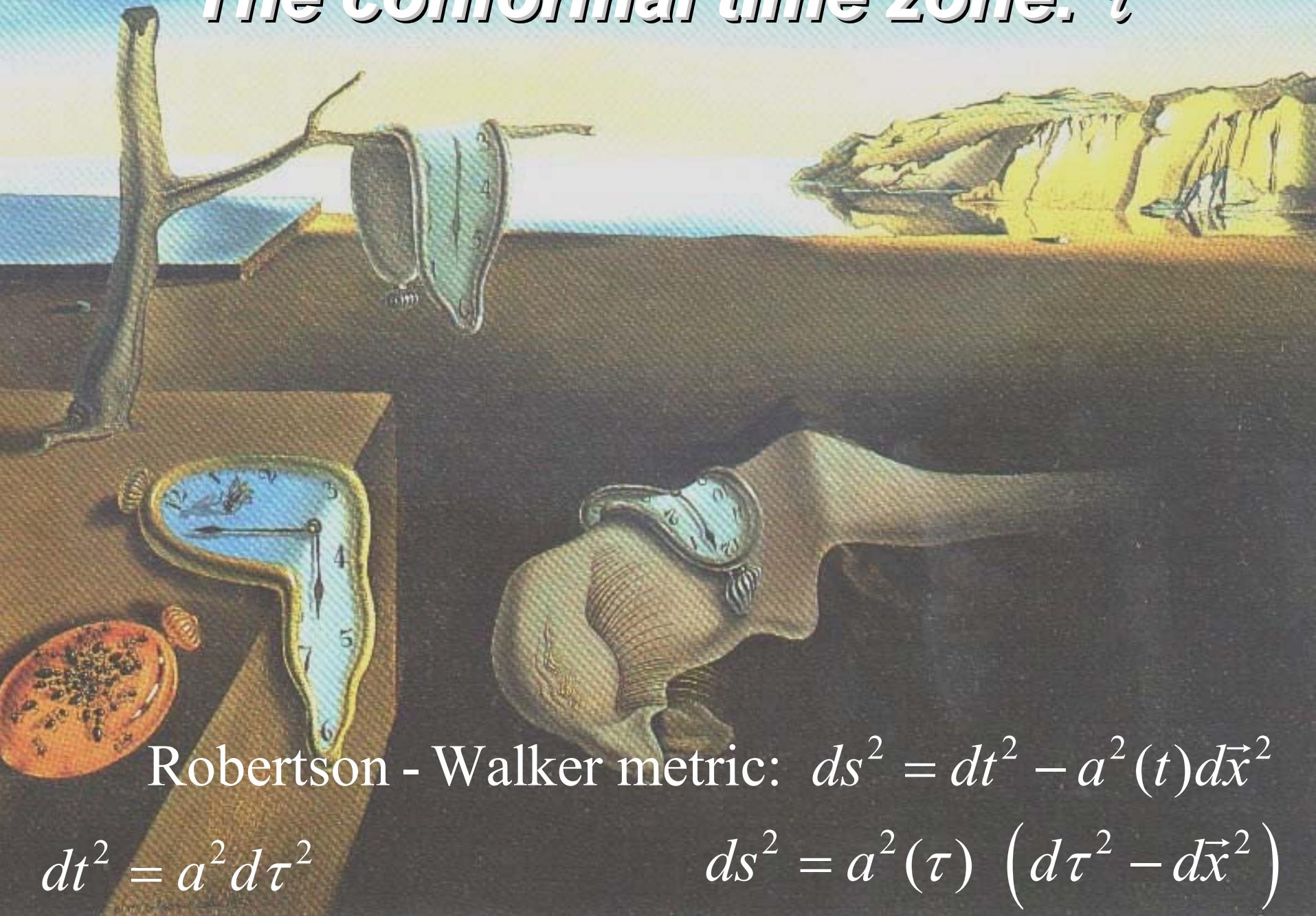
Scalar field X of mass M_X

Fourier modes [$a(\tau)$ = expansion scale factor]

$$X(\vec{x}, \tau) = \int \frac{d^3x}{(2\pi)^{3/2} a(\tau)} \left[a_k h_k(\tau) e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger h_k^*(\tau) e^{-i\vec{k}\cdot\vec{x}} \right]$$

(τ = conformal time)

The conformal time zone: τ



Robertson - Walker metric: $ds^2 = dt^2 - a^2(t)d\vec{x}^2$

$$dt^2 = a^2 d\tau^2$$

$$ds^2 = a^2(\tau) \left(d\tau^2 - d\vec{x}^2 \right)$$

Scalar field X of mass M_X

Fourier modes [$a(\tau)$ = expansion scale factor]

$$X(\vec{x}, \tau) = \int \frac{d^3x}{(2\pi)^{3/2} a(\tau)} \left[a_k h_k(\tau) e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger h_k^*(\tau) e^{-i\vec{k}\cdot\vec{x}} \right]$$

Mode equation (τ = conformal time)

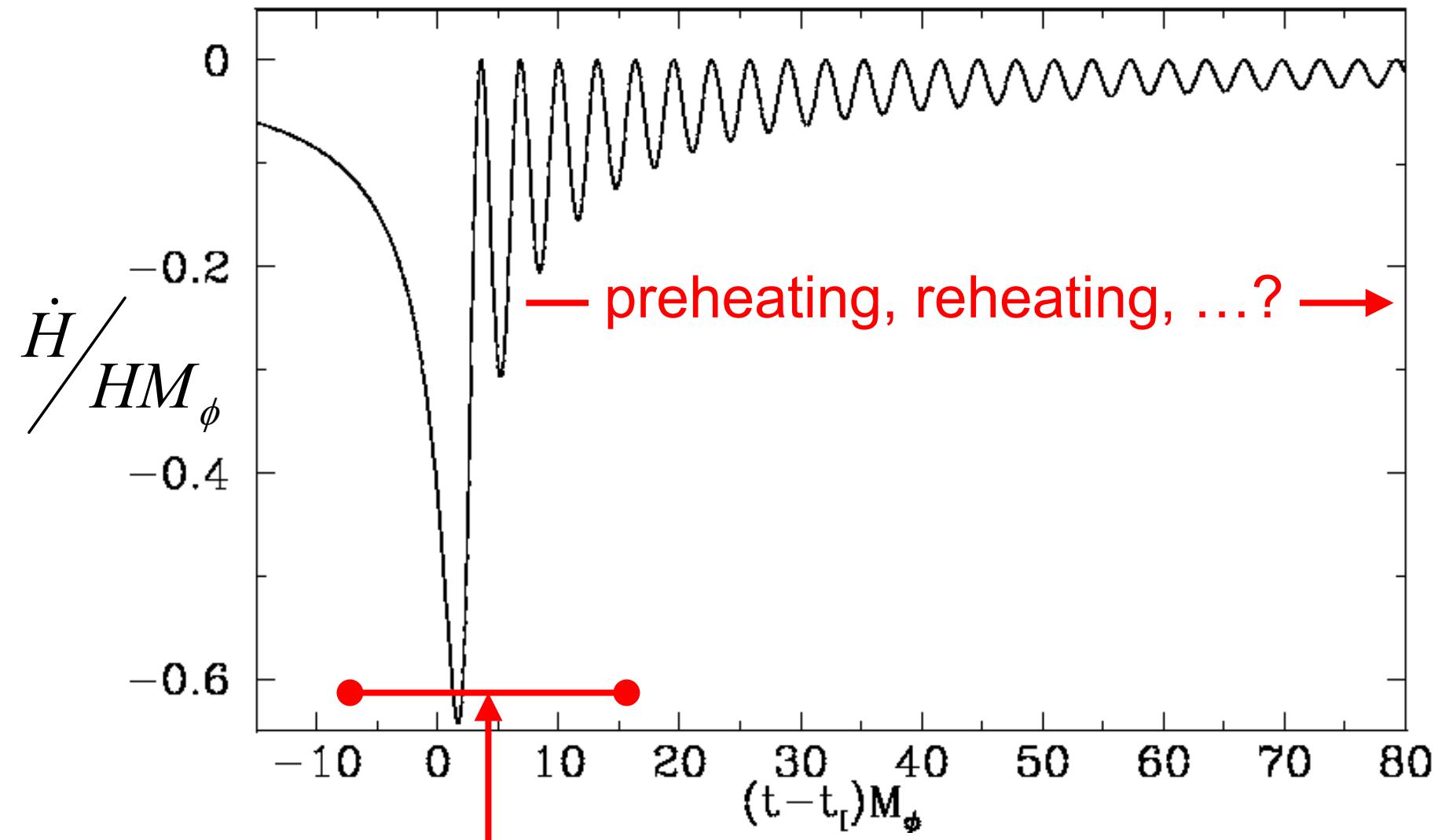
$$h_k''(\tau) + \left[k^2 + M_X^2 a^2 + (6\xi - 1) \overset{\text{0}}{\cancel{a''/a}} \right] h_k(\tau) = 0$$

$$h_k''(\tau) + \omega_k^2 h_k(\tau) = 0$$

Particle creation in nonadiabatic region

measure of nonadiabaticity $\propto \frac{\omega'_k}{\omega_k}$ or $\frac{\dot{H}}{H^2}$

Background fields in chaotic inflation



nonadiabatic region: particle creation

Particle production

No-particle state in past

$$a_k^0 |0\rangle = 0 \quad h_k^0 \leftrightarrow a_k^0$$

$h_k(\eta)$ adulterated as $\omega_k(\eta)$ changes

$$h_k = \alpha_k h_k^0 + \beta_k h_k^{0*}$$

$$a_k = \alpha_k a_k^0 - \beta_k a_k^{0\dagger}$$

Particle creation

$$N_k = \langle 0 | a_k^\dagger a_k | 0 \rangle \propto |\beta_k|^2$$

Particle production

Solve wave equation

$$h_k''(\tau) + \omega_k^2(\tau)h_k(\tau) = 0$$

$$\omega_k^2(\tau) = k^2 + M_X^2 a^2(\tau)$$

$$h_k^0 = 1/\sqrt{2\omega_k^0} \quad h_k'^0 = -i\sqrt{\omega_k^0}/2$$

Find Bogoliubov coefficient

$$|\beta_k|^2 = \frac{|h_k'|^2 + \omega_k^2 |h_k|^2 - \omega_k}{2\omega_k}$$

Number density proportional to

$$\int_0^\infty \frac{dk}{2\pi^2} k^2 |\beta_k|^2$$

Particle production

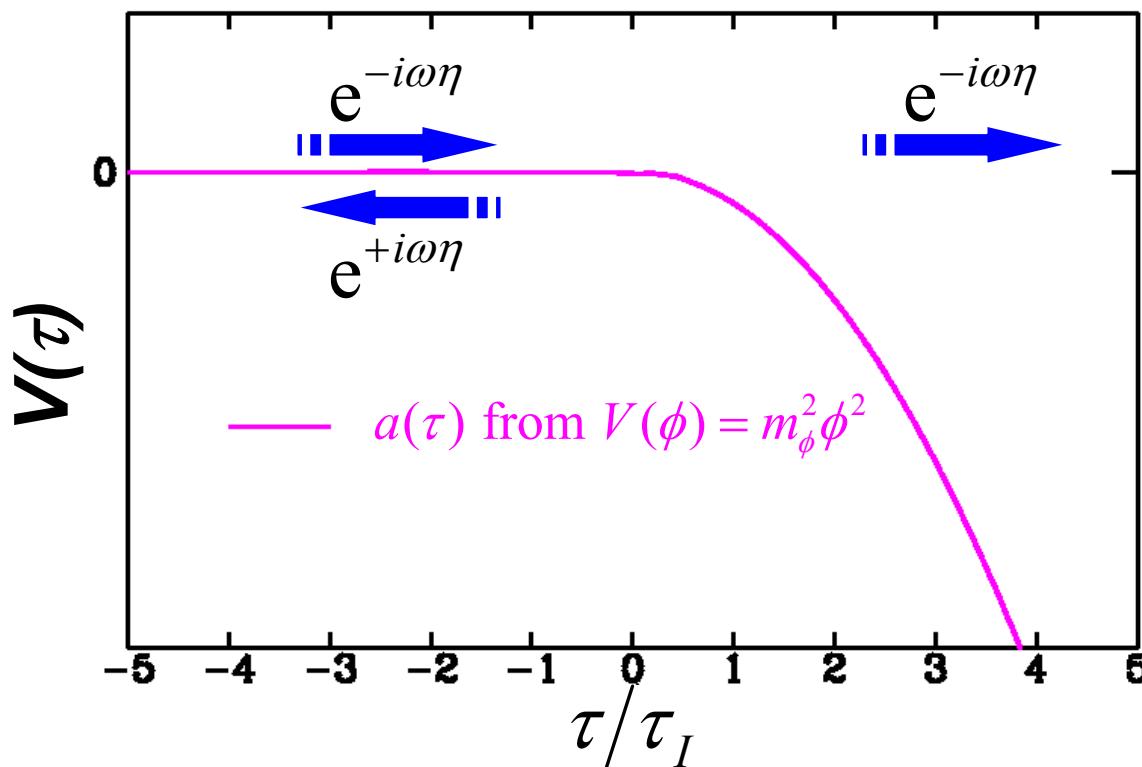
Mode equation:

$$h_k''(\tau) + \left(k^2 + \frac{M_X^2}{H_I^2} a^2 \right) h_k(\tau) = 0$$

$$\tau \rightarrow k; \quad h_k(\tau) \rightarrow \psi(x); \quad k^2 \rightarrow E; \quad \frac{M_X^2}{H_I^2} a^2 \rightarrow -V(x)$$

Wave equation:

$$-\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$



Particle production

Field theory : the result is almost always *infinite!*

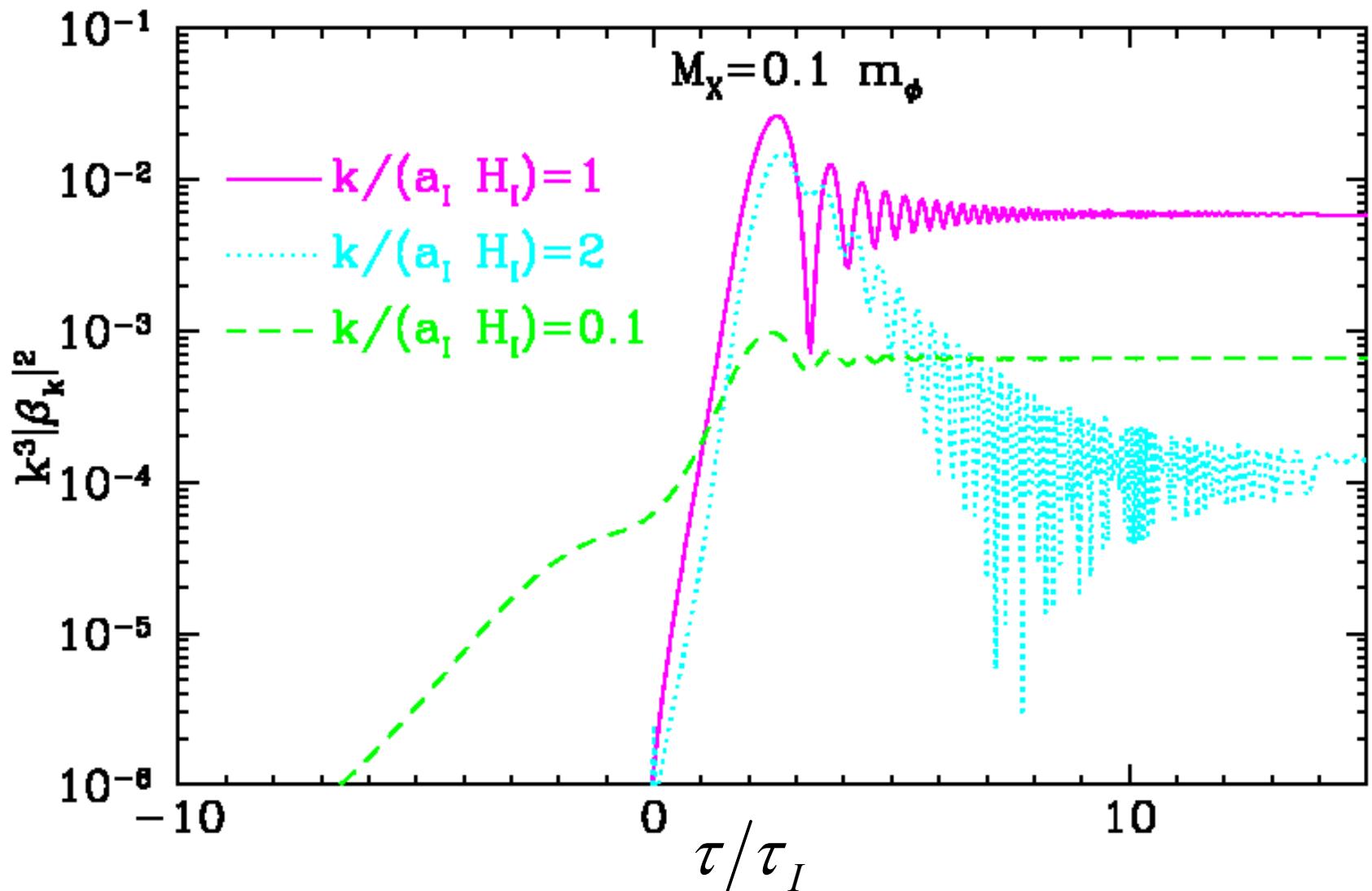
finite result if $\frac{d^n}{d\tau^n} \left[\frac{1}{a^2(\tau)} \frac{da^2(\tau)}{d\tau^2} \right]$ **finite**

inflation $a \propto e^{H_I t}$	$a^2(\tau) \propto \tau^{-2}$ $-\infty < \tau < 0$ $0 < t < +\infty$	fails at $\tau = 0$ $t = +\infty$
matter/ radiation $a \propto t^{2/3}, t^{1/2}$	$a^2(\tau) \propto \tau^2, \tau$ $0 < \tau < +\infty$ $0 < t < +\infty$	fails at $\tau = 0$ $t = 0$

start in inflation, end in matter/radiation

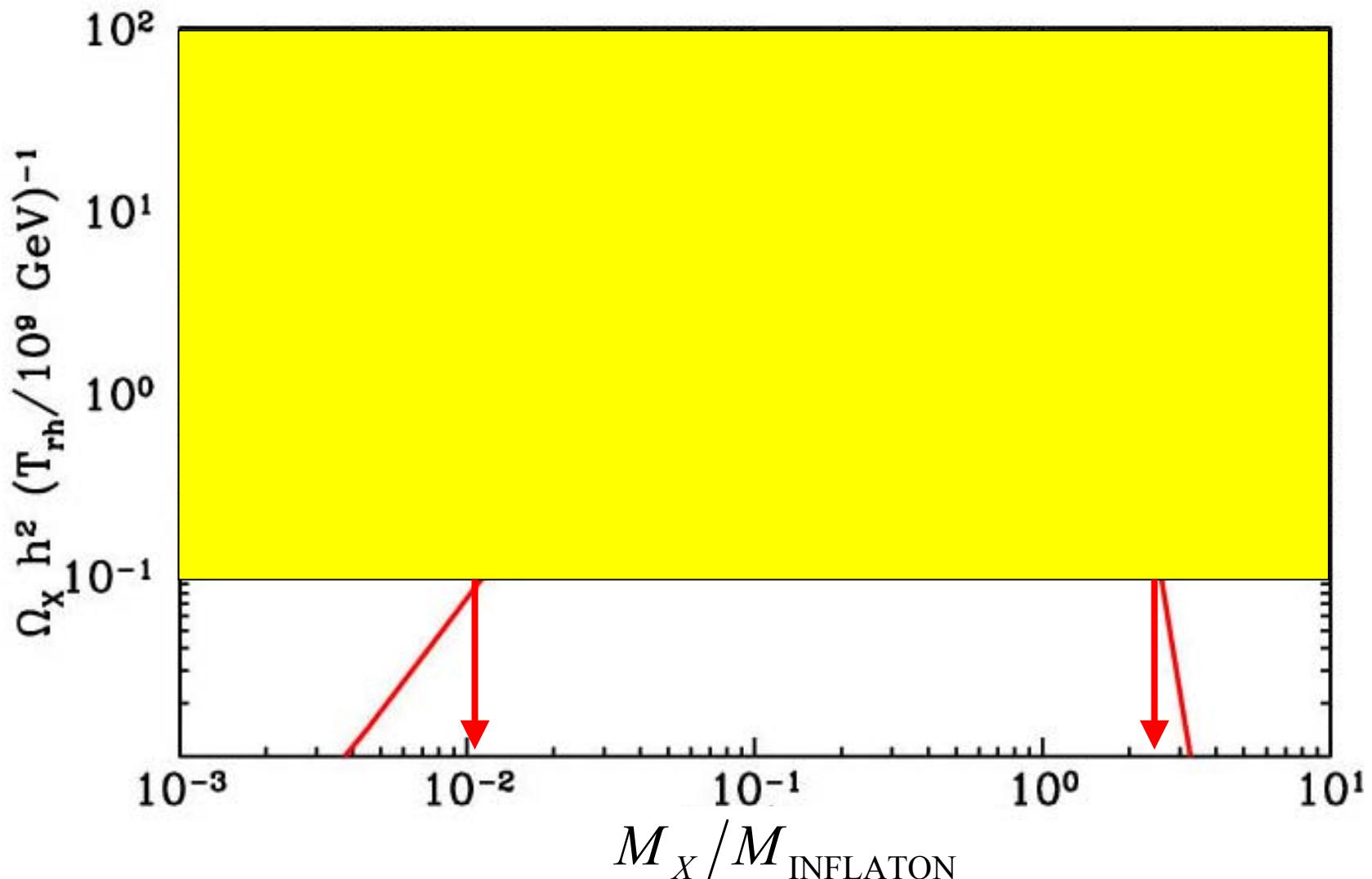
Particle production

$$\tau/\tau_I \sim 1 \quad k \sim aH$$



Particle production

Chung, Kolb, Riotto (also Kuzmin & Tkachev)



$\Omega_X \approx 1$ for $M_X/M_{\text{INFLATON}} \approx 1 \Rightarrow M_X \approx 10^{10} \text{ to } 10^{15} \text{ GeV}$

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